Weight Until Dark

In the beginning, there was the Dow. And it was good, or at least good enough for the proverbial government work. The venerable Dow Jones averages have always been price-weighted in their construction; the original reason being that it was too difficult in 1896 to calculate an index of 20 stocks any other way and still have time to publish it in an afternoon newspaper. True government work would have mandated that all of Wall Street remain in the office until a more complex index was published, revised, and leaked – although not necessarily in that order.

A price-weighted index is calculated by simply summing the prices of the stocks in the index, now numbering 30 for the Dow Jones Industrial Average, and dividing by a divisor which reflects the effects of stock splits and dividends. The DJIA's membership is determined by Dow Jones, best known as publishers of *The Wall Street Journal*, and as the components of the index change, the divisor has to change in order to maintain historical continuity.

$$DJIA = \frac{\sum_{i=1}^{30} P_i}{Divisor}$$

The S&P 500, on the other hand, is a capitalization-weighted index calculated by multiplying the number of outstanding shares for each of the 500 stocks by its price, multiplying by 10, and then dividing by the 1941-1943 baseline value. Membership in the S&P 500 is determined by Standard & Poors, and while it is roughly mechanical at the 500 largest companies by market capitalization, some discretion is allowed: The popularity of index funds in recent years gives new additions to the index an automatic increase in ownership and a concomitant boost in price, all else held equal.

$$S\&P500 = 10 * (\frac{\sum_{i=1}^{500} N_{i,t} * P_{i,t}}{OriginalValue})$$

Each stock in the S&P 500 has a weight. All of the stocks in the DJIA are members of the S&P 500, and a table of their weights – totaling nearly 25% of the S&P 500 – is given below.

Dow Jones Industrial Stocks' Weight In The S&P 500	
General Electric	3.303%
Coca Cola	2.290%
Exxon	1.894%
Merck	1.654%
Wal-Mart	1.569%
IBM	1.418%
AT&T	1.221%
Philip Morris	1.198%
Procter & Gamble	1.174%
Johnson & Johnson	1.156%
Travelers Group	0.860%
Walt Disney	0.801%

DuPont	0.785%
Hewlett Packard	0.638%
Chevron	0.593%
American Express	0.578%
General Motors	0.547%
McDonalds	0.511%
Boeing	0.434%
3M	0.337%
Eastman Kodak	0.295%
Allied Signal	0.276%
J.P. Morgan	0.251%
United Technologies	0.251%
Sears Roebuck	0.223%
Caterpillar	0.204%
International Paper	0.157%
Alcoa	0.142%
Goodyear Tire	0.109%
Union Carbide	0.073%
Total:	**24.942%**

The mechanics of the two indices creates different price behavior. If the price of General Electric and Union Carbide, the largest and smallest stocks in the DJIA each rise by $1.00 per share, then each stock will have an equal effect on the DJIA. However, in the S&P 500, the same $1.00 per share increase will have a [3.03% ÷ 0.073%], or 45.25 times greater contribution to the index's price change for General Electric as it will for Union Carbide. It should come as no surprise that for most of the bull market of the first seven months of 1998, the S&P 500 was propelled higher by strong gains in a few of its largest-weighted stocks, such as Microsoft, Pfizer, and Lucent Technologies – none of which are DJIA members.

Another important and irreducible difference between the two indices derives from dividends. Both corporate America and the investing public have accepted a far-lower dividend yield during the 1990s bull market than prevailed in previous generations. It was common during the 1930s and 1940s for dividend yields to exceed bond yields as compensation for the greater risks of equity investing. However, while bond coupons are deducted against corporate income, dividends are taxed twice, once as corporate profits and once as ordinary income to the investor. As a result, investors have chosen the far-riskier course of eschewing dividends in favor of hoped-for capital gains; while this has been a wise course of action since 1982, bull markets are not forever, and investors may one day regret the lack of immediate income on their equity holdings.

The DJIA pays a higher dividend yield than does the S&P 500, 1.71% to 1.48% as of August 7, 1998. Since the value of a stock index future is

$$Future = Index*(1+r) - \sum_{i=1}^{N} Div_i*(1+r_i)$$

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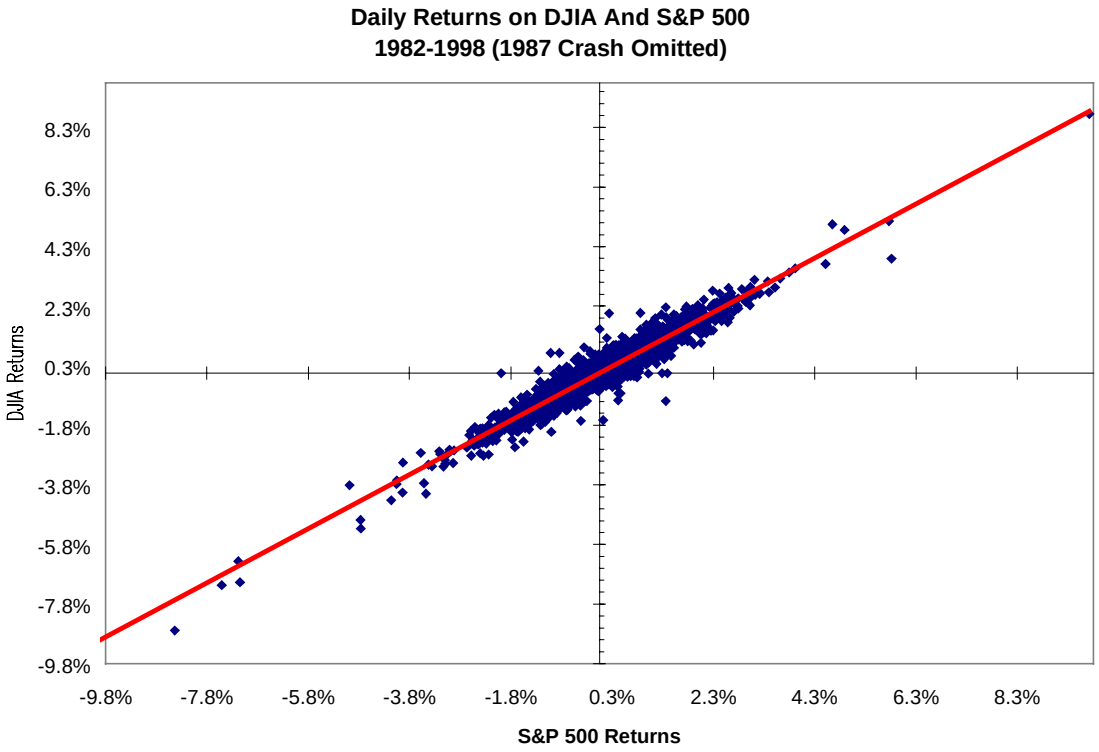
the convergence of the future to the underlying cash index must be smoother for the S&P 500 than for the DJIA, since the S&P basis path conforms more closely to an interest rate curve. In addition, the ex-dividend price adjustment for the DJIA will be greater than for the S&P 500 due to the effects of price weighting.

Even with all of these considerations, the two indices have tended to track each other closely over the years. A comparison of the two sets of daily returns since just after the 1982 introduction of S&P 500 futures shows a very tight relationship; the October 1987 crash is omitted from display in order to preserve scale. The regression equation is

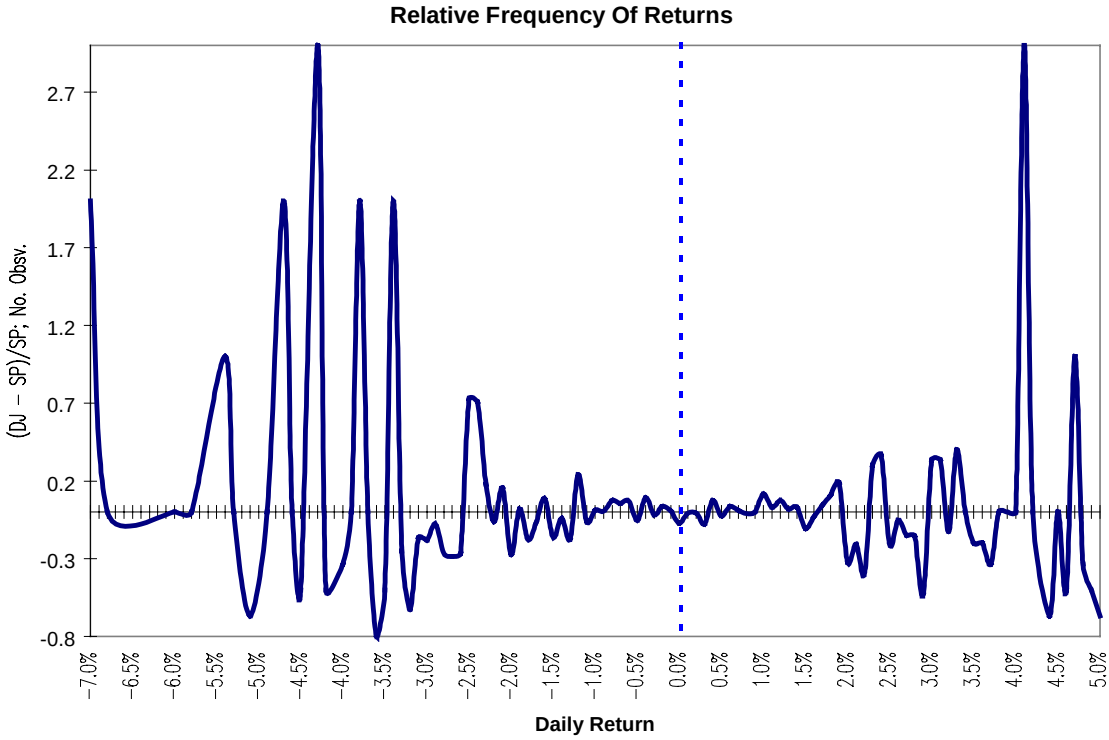
DJIA Return = 1.0212 * SP500 Return; R^2 = .9278



The regression coefficient of 1.0212 tells us that the DJIA is more volatile than the S&P 500, which should be expected given its narrower membership base and, more importantly, its price-weighting which should both exaggerate moves on the extreme days in the stock market and produce more non-event days. If this is so, then the kurtosis of the distribution of returns, produced by the formula below for all observations between 1934 and 1998 to-date, should be higher for the DJIA than for the S&P 500, and it is, 43.76 to 30.38.

$$\left\{\frac{N(N-1)}{(N-1)(N-2)(N-3)}\sum_{i=1}^{N}\left(\frac{X_i-\mu}{\sigma}\right)^4\right\}-\frac{3(N-1)^2}{(N-2)(N-3)}$$

This heavily-peaked distribution for DJIA returns could be of use in trading if it differed significantly from that of the S&P 500. Once again, we can take the 1934-1998 sample, construct a histogram of returns at 0.1% intervals, and compare the relative frequency of each market in each return slot. The results, plotted over the spectrum of daily returns from -7.0% to 5.0%, indicate a greater frequency of large returns for the DJIA, especially for large negative returns.



Unembedding Embedded Options

The nature of these different distributions suggests that selling a DJIA position against a S&P 500 position is equivalent to owning a down-and-in barrier put option at a -2.0% daily return, and to being short an up-and-in barrier call option at a 2.0% daily return on the equity market (see "Why Be Average," *Futures*, October 1997). Since the average daily return on the DJIA and S&P 500 since 1934 has been .0273% and .0287% per day, with daily standard deviations of return of .9191% and .9361%, respectively, we are talking about barriers of more than two standard deviations per day, or well more than 95% of total observations, remaining inside the barriers.

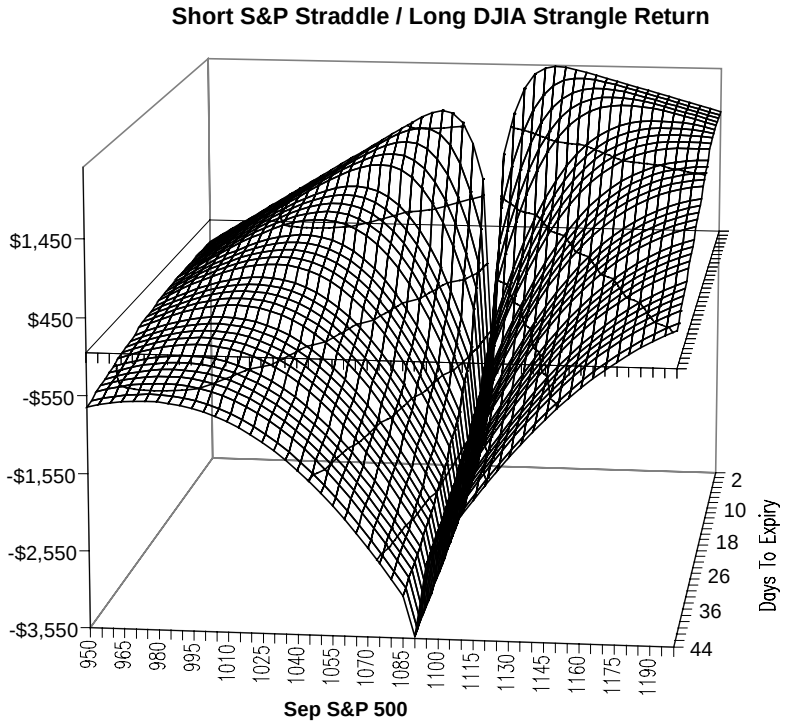
S&P 500 and DJIA futures have different underlying values, (see "The People's Stock Index Futures," *Futures*, March 1998) and these must be accounted for in designing spreads. On August 5, 1998, the September DJIA future had an underlying value of $86,050, while the September S&P 500 had a value of $272,125, or 3.16 times as much. While many might be loathe to sell three DJIA futures against one S&P 500 future, and while this could create some unwelcome margin problems, we should point out that even on a big down day such as August 4, 1998, the gain on three short DJIA futures would have been $9,180 against a loss of $10,900 for one long S&P 500.

A more efficient capture of the optionality embedded in the spread between the two indices can be effected, unsurprisingly, by using options. Using data from the close of business on August 5, 1998, when the September S&P 500 and DJIA futures settled at 1088.50 and 8605, respectively, the following trade is suggested:

Sell Sep S&P 500 1090 straddle at $60.2 for a credit of $1,505
Buy 3 Sep DJIA 8500 / 8700 strangles at $50.75 for a total debit of $1,523

The 8500 put strike and 8700 call strike for the DJIA futures reflects barrier nature of the trade; these levels represent strikes at slightly more than 1.0% from at-the-money levels. The total delta of the short S&P 500 straddle is -.018, while the total delta of the DJIA strangle is .111, for a net

delta at initiation of .093. We can depict the path of this trade across the dimensions of price and time to expiration with a constant 1.0212 elasticity between the two indices.



The trade's slight positive delta at initiation gives it a slight upward bias. The loss zone becomes increasingly narrow as expiration approaches due to the positive impact of time decay on the net spread; the maximum loss at expiration in a perfectly static market is a little more than $2,100 per spread.