Limitations Of Chaos

The human mind rejects randomness. We are trained for our earliest days to search for orderly patters in the world around us, organize the information into neat categories and construct rules ("models") describing what we see. We naturally tend to place ourselves at the core of newly described systems.

But most scientific history consists of a gradual removal of man as the focal point of our self-described systems: Copernicus removed the Earth from the center of the universe, Newton added mechanical laws to nature, Lyell and Hutton demonstrated that the rocks beneath our feet are not fixed.

Most important, the great physicists of the twentieth century - Einstein, Planck, Heisenberg, Bohr - described the incomprehensible strangeness of the seemingly orderly world around us through the theories of relativity and quantum mechanics.

Upholding this tradition is the intellectual revolution called chaos theory: The more we know in an objective sense, the more mysterious, spiritual and ultimately unknowable we find the world to be.

Hallmarks

Chaos theory satisfies the acid test of any worthwhile idea: You can describe it simply enough to fit on a T-shirt ("order in disorder"). Complex systems generally can be described with a few non-linear equations. Repetitions of those equations produce results that can be analyzed by plotting one phase of the process against others, a function against its derivatives. Instead of the traditional price-time axes for market data, for example, the x-axis might plot five-day momentum and the y-axis 10-day momentum.

Financial data commonly are converted into some scale-independent statistic, such as logarithmic return, $\ln(P_t/P_{t-1})$. A more powerful tool for generating "phase descriptors" of this type involves a process called "embedding." Although market activity typically is shown in a two-dimensional chart format, price vs. time, in reality it operates in several dimensions, such as price, time, volume, momentum, acceleration, etc. Embedding a series involves a separator (S) and a final embedding dimension (D). A single time series like the S&P 500 can be converted into a series of vectors ($X_t=X_t,X_{s+t},X_{2s+t},X_{3s+t}...X_{Ds+t}$) that define the various dimensions of the system.

In practice, a "D" of five or greater indicates a random series, which may explain why so much chaos-based trading research has produced so few results. For a series to be tradable, it should be embedded in as few dimensions as possible.

The embedded data often exhibits scale-independent, self-similar patterns called fractals that hint at an underlying order. Most plots of natural process converge upon a final state. For example, the temperature of a cup of coffee plotted against dimensions such as time or the rate of cooling will converge toward that of the room. In chaos parlance, this is a "point attractor."

If, however, we are dealing with an economic system whose values do not converge, but rather oscillate about various regions on the graph, then we have discovered a "strange attractor." Many of these look like figure eight's with several tracks on each loop of the eight. Strange attractors in practice are produced by a search over increasingly higher embedding dimensions until further repetitions fail to change the structure of the phase plot, i.e., a pattern appears. Such patterns are non-linear representations of some kind of order in the series.

The dimensions used to describe the series often are referred to as "phase space." A number called the Lyapunov exponent, which describes the rate at which the data diverge, measures the sensitivity of the attraction process to the initial conditions of the system. A lexicon of chaos terms is offered below.

Chaos: The irregular behavior of a simple deterministic equation. A deterministic equation is one in which all parts are known, e.g., $X_{n+1}=4X_n*(1-X_n)$.

Bifurcation: The points at which the regular output of an equation becomes irregular.

Phase space: Data plotted against transformations of itself, e.g., X against its first or second derivatives.

Dominant frequency: Cycles per unit of time, where the unit of time is the interval between data points. Can be determined by such methods as Fourier transforms or maximum entropy analysis.

Lyapunov exponent: The rate at which trajectories in phase space diverge. This number will be positive for chaotic data.

The appeal of search for a chaotic explanation of a complex system like a market is twofold. First, our minds demand that we find an underlying order, and if we can do it with a few simple non-linear equations, as opposed to a large family of partial differential equations, so much the better. Second, visual inspection of a well-defined strange attractor leads to the conclusion that answers can be predicted for any known value of the independent variable. That is, you can predict the series.

Chaos And Economics

So extreme is the desire to forecast prices, trading firms spend large amounts of time and money crunching numbers, and, in the face of massive evidence indicating the imprudence of doing so, take positions based on them. Witness trading behavior on government report days as Wall Street reacts in shock and disbelief that its economists were wrong.

Yet simple logic dictates an economic system with numerous feedback loops, gaming strategies, expectations, technological change and external shocks is constantly attracted to a set of equilibrium levels whose movements in turn are influenced by the speed and efficiency of the attraction process.

The strange attractors in an economic system are embedded in a large number of dimensions, one of which is the collective expectations of the market participants: A change in expectations for tomorrow will alter decisions made today, which ultimately modify future expectations and decisions, etc.

But if the economy is at all sensitive to changes in initial conditions, (one of the characteristics of a chaotic system) then even highly accurate knowledge of nearly all dimensions of the economy will not necessarily lead to more accurate forecasts. This is the cruelest irony of all, because if such forecasts were possible and acquired credibility, their influence on both current decisions and expectations would change both the previous dimensions and the initial conditions of the system, rendering their predictive value worthless. Market feedback contains the seeds of its own destruction.

A parallel from the world of physics lies in the Uncertainty Principle, which states it is impossible to know both the position and momentum of a particle exactly; to observe one, the other must remain constant. As a result, the state of the particle is changed by the mere act of observing it, and the cumulative effect of these changes in all of the initial conditions produces vast and unpredictable effects on the ultimate course of the system.

A financial market corollary to the Uncertainty Principle is illustrated by the mere acts of analyzing and trading a market: This process changes the initial conditions of the system, and therefore makes it impossible to know all the dimensions of a market exactly. Without knowledge of the independent variables, (the embedding dimensions) we cannot forecast the dependent variable, price.

Modeling Approaches

Despite the formidable challenges presented above, research continues, including "brute force" efforts based upon massive tick-by-tick databases and sophisticated time series analysis models of the autoregressive conditional heteroscedasticity (ARCH) class.

Several models were presented at a recent conference sponsored by statistical economists Olsen and Associates. These models are excellent at breaking price series into trend and cycle components, but are weak in finding structure in the leftover random, or residual, component of the series. Unfortunately, it is in this statistical "noise" that chaos lies.

Still, we must recognize the limitations of chaos theory for financial applications. Fractals are patterns by another name. What is the difference between an embedding dimensions and the independent variables that have been used for decades by econometric models? It is unlikely there will ever be a viable forecasting tool based upon chaos theory; more promising are systems that borrow chaotic concepts and combine them with robust rule generators like genetic algorithms.

Artificial Life

Just as we can project the chaos and uncertainties found in physics onto economic data, we can project the adaptive resilience and robust evolution of biology into the creation of trading models. The chaotic model of market behavior as a "natural" system makes this approach a logical way to transplant chaos-based ideas like fractal patterns of embedding dimensions into a trading environment. Also, the creativity exhibited by living systems in adapting to each other and to their environment should be imitated by traders.

Genetic algorithms duplicate the living systems survival process in a computational environment: A large set of potential trading rules is included at the start of the process, the "genes" to be passed on. The success of these groups against some target objective, say profit maximization or return/risk maximization is recorded. Then comes a "mating" process, where the genes of successful organisms are somewhat randomly combined. In relatively few trials, the successful trading rules are combined into a system.

Such adaptive systems adjust to the changing landscape of chaotic market systems continually altered by multi-dimensional feedback and outside disturbances. Instead of trading a static market model, these systems trade the shifting characteristics of dynamic markets: The bond market you traded last year may not be the bond market you trade today.

But just as chaos theory is limited as a predictor, artificial life is limited as a system generator. Remember, it is estimated that over 99% of all species that ever existed are extinct (and if certain groups ever get ahold of nuclear weapons, cockroaches and pork belly locals will be the only survivors. Also like nature, new trading theories and technologies constantly compete to create new trading systems in the ongoing process of natural (artificial?) selection.