

## On And Off, In And Out, Up And Down...

Trader indecision, while common, is a perfectly ludicrous state of mind. After all, the market can only go up or down, and how difficult can that be? The ability to parse seemingly complex decisions into bite-sized yes or no decisions has led to the greatest explosion of economic value in history: Digital computers process information in a binary world of 1's and 0's.

Not only can we decompose large-scale complex problems in this manner, we can decompose all financial instruments into a set of binary payoffs or options as well. A binary option, often referred to as a digital option, pays off if a condition is satisfied. The payoff can be cash-or-nothing or asset-or-nothing. In the cash case, the holder will receive a predetermined cash payment if the option's terms are satisfied, while in the asset case, the holder will receive the underlying asset at the present value of its terminal price. The difference between the cash and asset settlements is a gap option.

For example, a cash-or-nothing binary call will pay off if it settles in the money with the magnitude of [underlying - strike] being irrelevant. If soybeans settle at \$5.20 per bushel on the option expiration day, then the value of an American \$5.00 call option will be  $\$5.20 - \$5.00$ , or \$0.20. We can restate this as a package of 20 \$5.00 binary calls each with a payoff of \$0.01, or as any other combination of binary calls with a total payoff of \$0.20.

As has been the case with all of the exotic options we have examined in this series, this payoff structure matches commercial reality well. Consider an electric utility faced with a weather-related decision (see "A Matter of Degrees," *Futures*, June 1999). If the temperature exceeds 92°, then they will have to bring a peaking plant on line, and that will involve a package of minimum operating expenses. These costs, for the most part, will not rise along with the temperature, so the magnitude of excess is irrelevant. If, however, the costs do rise, the binary option solution is simple: Buy call options with payoffs beginning with temperatures greater than 92°.

### Binary Flavors

Barrier options, which are akin to stop orders, (see "[Poisson In The Wind](#)," *Futures*, November 1998) are among the most common binary options. They include:

- **Up-and-in.** Like entering a new long on a buy stop, option becomes active when the underlying rises to the barrier price.
- **Up-and-out.** Like exiting a long position at a target, option becomes inactive when the underlying rises to the barrier price.
- **Down-and-in.** Like entering a new short on a sell stop, option becomes active when the underlying falls to the barrier price.
- **Down-and-out.** Like exiting a short position at a target, option becomes inactive when the underlying falls to the barrier price.

Of course, this being the world of exotic options, we cannot stop at such a simple point. We can settle these options at expiration, or we can make them "one-touch." We can make the barrier knock in or out gradually; this is referred to as a soft barrier. If a barrier contract provides for a set of minimum payoffs if a certain intermediate levels are hit, we call it a ladder option. And, of course, we can have double barrier options.

We can make two-factor barriers where an option on one commodity knocks in or out based upon another commodity's value. For example, an industrial fuel buyer might wish to extinguish a call option on natural gas if the price of residual fuel oil falls below a certain price. Or, a fast-food franchise might wish to extinguish a call on chicken if the price of beef falls below a certain price.

We can combine barriers into range binaries, which pay off if the market stays within a stipulated range; this is akin to a short strangle with plain vanilla options. These barriers or ranges can have rebates, or

partial return of the premium if they did not pay off to the holder (this is the sort of thing you'd lose your license over in the exchange-traded world). Much of the value of a binary barrier option derives from these rebates: A down-and-out or down-and-in call without a rebate is a standard call option; the same applies to the up-and-in or up-and-out situation for a put option. We invert the range binary into a limit binary, which pays off if both sides of the range are hit. Or, we can combine range and limit binaries into a boundary binary, which has one payoff if both sides are hit, and another payoff if neither side is hit.

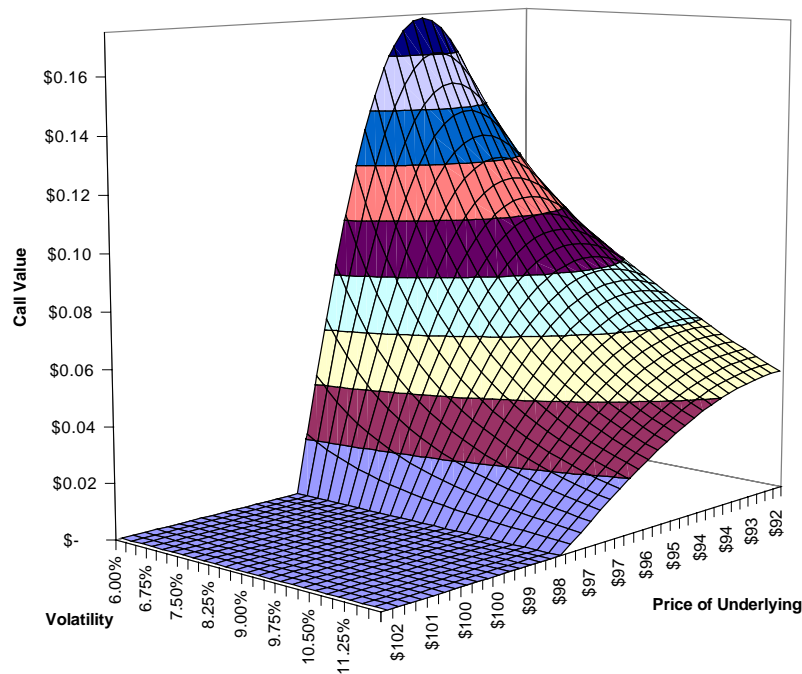
Further variations on this theme include wall options, which pay off a specific amount for every day the market stays above or below a specific barrier price, and knockout walls, which pay off until an extinguishing barrier is reached. If we add a second wall, we get a corridor option, which pays off for every day the market stays within the range. While the commercial utility these range options may not be apparent immediately, their payoff structure recognizes the avoided costs one of the parties will enjoy by virtue of the market staying above or below a certain price or within a certain range.

Since binary options are the fundamental building blocks of finance, akin to programming a computer in a very low-level language such as Assembler, it is easy to get carried away with building complex structures just because you can. It is critical to verbalize the problem first, and then work your way backward to the most precise trade (see "[Compounding The Problem](#)," *Futures*, May 2000).

### Binary Behavior

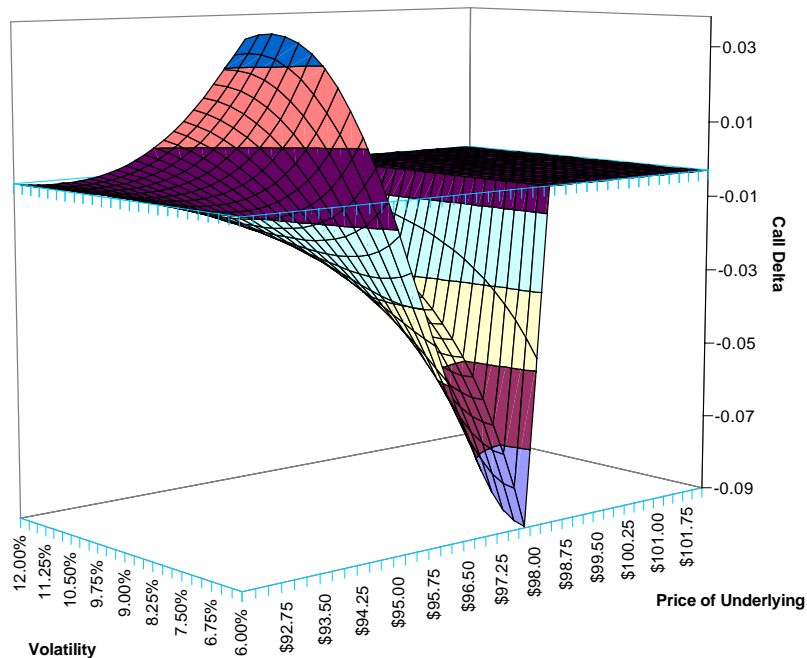
The discontinuous payoff structure of a binary option produces some odd behavior. Higher volatility, for example, increases the probability of a barrier being hit over the option's lifetime, and this will reduce the value of up-and-out and down-and-out barriers, as seen in the chart below for a Treasury bond binary barrier call with four months left to maturity. The same effect obtains with increased time to maturity; the longer option life increases the probability of the option becoming extinguished.

Price Of An Up & Out Call With \$98 Knock-Out, \$96 Strike



The delta of an up-and-out call will rise along with the price of the underlying, but then it must become negative as the barrier is approached: The value of the call will become zero at the barrier, and therefore the option's price with respect to the underlying will decrease. The delta of the option will pass through zero as well, implying a zone of no hedge at all. As the time to maturity decreases, the delta of the up-and-out call will decrease as well, as seen below for the same four-month Treasury bond call. Here again, the delta of the call will pass through zero and then take on negative values.

**Delta of an Up & Out Call With \$98 Knock-Out, \$96 Strike**



The counterintuitive effects of volatility and time on binary barrier option prices and the very real occurrence of negative deltas for long call options and positive deltas for long put options create some interesting problems and opportunities for those maintaining a portfolio of these instruments. For one, hedge a binary with a vanilla is a true oil and water situation. Binaries and barriers must be hedged with other binaries and barriers, and this includes all of their sub-varieties such as ranges, walls, corridors, etc.

At this point, many may wonder if the exercise is worth its complexity, and the answer here is a resounding "Yes." Let's return to a point we've made throughout this series, that exotic options match commercial needs and trading applications far better than do vanilla options. For speculators, they are cheaper and have greater leverage. For hedgers, they can be sculpted to meet any set of contingent economic constraints. In all cases, we can obtain the exact payoff matrix we want at the price we are willing to pay and at the risk level we are willing to take.

### **Concluding Thoughts**

The genesis of this series was the impending reality of an electronic trading world in which we all can trade whatever instruments exist without the artificial restrictions channeling our talents into a small number of standardized instruments. This will be a very strange world at first, and many of us will not survive the transition. So be it: Markets create winners and losers, and that was the bargain we accepted. But economic history is clear on one count, as we lower the cost of doing business and as we increase the number of choices available, demand explodes. After all, demand for transportation increased during the transition to automobiles from horses, and demand for information certainly increased during the various

stages of the computer revolution. Demand for risk management, asset allocation, and trading acumen will increase, and increase exponentially, in our new world. The cost of participation is preparation, and hopefully this series has been a step in that direction.